ALLOWANCE FOR THE EFFECT OF NONISOTHERMICITY ON HEAT TRANSFER IN CHANNELS DURING THE LAMINAR MOVEMENT OF LIQUIDS HAVING A LINEAR LAW OF FLUIDITY

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Nonisothermal heat exchange during the laminar flow of liquids having a linear law of fluidity in the thermal initial section of channels is analyzed. The calculation for the condition that t_ω=const takes into account the effect of the temperature on the zero-point fluidity and the coefficient of instability of the structure when the thermal conductivity (diffusion) depends on the shear stress.

The problem of the effect of the nonisothermicity of a stream on the local heat exchange characteristics during the flow of non-Newtonian liquids in the initial section of channels is very complicated. Because of the nonlinearity of the original equations the estimates of the effect of nonisothermicity are based on the results of numerical calculation [1]. It is desirable to have equations, if only approximate, reflecting the effect in explicit form.

The main purpose of the present report is to take into account the dependence on the temperature and the tangential shear stress (τ) of the effect of the fluidity (the reciprocal of the viscosity $\mu = 1/\varphi$) on the principal dynamic and thermal characteristics of the laminar stream in the thermal initial section of channels. An established velocity profile at the entrance to the channel and a constant wall temperature along its length are assigned in this case. It is also assumed that the Prandtl numbers of the liquids under consideration are much larger than unity and that their coefficient of thermal conductivity can depend on τ. The calculations performed below remain in force for the study of effects in the diffusional initial section of channels. In this case the fluidity is set up as a function of τ and the concentration of the substance.

The calculation is conducted for rheological liquids (including the case of ordinary liquids with $\Theta \approx 0$) which obey a linear law of fluidity [2]

$$-\frac{\dot{W}}{\tau} \equiv \varphi(\tau, t) = \varphi_0(t) \pm \Theta(t)|\tau|, \tag{1}$$

where $\varphi_0 = 1/\mu_0$ is the fluidity as $\tau \to 0$, Θ is the coefficient of instability of the structure of the liquid with increasing (decreasing) fluidity, τ is the tangential shear stress, $\dot{W} = \partial\omega_x/\partial r$ is the velocity gradient, t is the temperature of the liquid, and r is the radius of the tube.

The data of [3, 4] indicate the small (10-30%) effect of the shear velocity on the processes of heat conduction. However, there are reasons to assume [5] that the shear velocity can have a more important effect on diffusional effects connected with mass transfer of the substance.

For small values of τ one can adopt the following interpolarion equation for the coefficient of thermal resistance (or diffusion) [6]:

$$\lambda^{-1}(\tau) = \lambda_0^{-1}[1 - \lambda_0 n_\lambda |\tau|], \tag{2}$$

where λ_0^{-1} is the coefficient of thermal resistance as $\tau \to 0$.

To solve the problem Eqs. (1)-(2) in dimensionless form with certain estimates and assumptions [1] are combined with the nonisothermal condition of equilibrium

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$$\frac{1}{1-Y}\frac{\partial}{\partial Y}\left[(1-Y)\frac{1}{\tilde{\varphi}}\frac{\partial W_X}{\partial Y}\right]=f(X),$$

$$f(X)=\int_0^1\left[\frac{\partial\tilde{P}}{\partial X}\frac{\mathrm{Re}_{00}}{2}+\frac{\mathrm{Re}_{00}}{2}\left(W_X\frac{\partial W_X}{\partial X}+W_Y\frac{\partial W_X}{\partial Y}\right)-\frac{\partial}{\partial X}\left(\frac{1}{\tilde{\varphi}}\frac{\partial W_X}{\partial X}\right)-\frac{\partial\tilde{\varphi}^{-1}}{\partial X}\frac{\partial W_X}{\partial X}-\frac{d\tilde{\varphi}^{-1}}{\partial Y}\frac{\partial W_Y}{\partial X}\right]dY, \tag{3}$$

$$\left(\frac{1}{\tilde{\varphi}}\frac{\partial W_X}{\partial Y}=\tau^{XY}\right),$$

the condition of continuity

$$\frac{\partial W_X}{\partial X}+\frac{1}{1-Y}\frac{\partial}{\partial Y}[(1-Y)W_Y]=0, \tag{4}$$

the condition of constancy of the flow rate

$$\int_0^1 W_X(1-Y)\,dY=-0.5, \tag{5}$$

and the equation for heat and mass transfer

$$0{,}5\,\mathrm{Pe}_0\left(W_X\frac{\partial\upsilon}{\partial X}+W_Y\frac{\partial\upsilon}{\partial Y}\right)=\frac{1}{1-Y}\frac{\partial}{\partial Y}\left[(1-Y)\tilde{\lambda}^{-1}(\tau)\frac{\partial\upsilon}{\partial Y}\right]. \tag{6}$$

In Eq. (6) υ is understood as the dimensionless temperature or concentration of the substance, while $\lambda(\tau)$ is the respective coefficient of thermal conductivity or diffusion. Here $Y\equiv y/R=1-(r/R)$, $X=x/R$, $W_{X,Y}=\omega_{x,y}/\langle\omega\rangle$, $\upsilon=(t-t_0)/(t_\omega-t_0)$, $\mathrm{Pe}_\Theta=\langle\omega\rangle\,2R/a_0$, $\mathrm{Re}_{00}=\langle\omega\rangle\rho\varphi_{00}2R$, $\tilde{\varphi}=\varphi/\varphi_{00}$, $\tilde{P}=P/\rho\langle\omega^2\rangle$, $\tilde{\lambda}(\tau)=\lambda_0/\lambda(\tau)$. For convenience we define $\tilde{\tau}=\tau/\tau_\omega$, $\Delta=y_T/R$, $K=\lambda_0 n_\lambda|\tau_\omega|$, where Δ is the dimensionless thickness of the thermal (diffusional) boundary layer, τ_ω is the shear stress at the wall of the channel, and φ_{00} and Θ_0 correspond to t_0 (the temperature at the exit).

Calculations of heat exchange by the integral method are the most common at present. In the present work the solution of the system of differential equations (1)-(6) for the boundary condition $t_\omega=\mathrm{const}$ is determined by the method of successive approximations.

In the first approximation we consider the case of $\omega_y\ll\omega_x$. As a consequence of this approximation $\partial\omega_X/\partial x\approx 0$, $p\neq f(y)$. It is seen from Eq. (3) that

$$\tilde{\tau}=1-Y.$$

In this case the expression for τ_ω is known [2]:

$$\tau_\omega=\frac{5}{8}\frac{\varphi_0}{\Theta}\left[\left(1+\frac{128\beta}{5\,\mathrm{Re}_0}\right)^{0,5}-1\right],\quad \beta=\frac{\Theta}{\varphi_0}\rho\langle\omega\rangle^2.$$

Consequently,

$$\tilde{\lambda}(Y)=1-K(1-Y).$$

Equating the left side of Eq. (6) to some constant and allowing for the fact that $\partial\upsilon/\partial Y=0$ and $\upsilon=0$ when $1-Y=0$ and that $\upsilon=1$ when $Y=0$, we find

$$\upsilon=\left(1-\frac{2}{3}K\right)^{-1}\left[(1-Y)^2-\frac{2}{3}K(1-Y)^3\right]. \tag{7}$$

Assuming that in a moderately wide temperature interval a binomial dependence of the type

$$\tilde{\varphi}_0\equiv\frac{\varphi_0(\upsilon)}{\varphi_{00}}=1+\psi\upsilon,\ \text{where}\ \psi=\frac{\varphi_{0\omega}}{\varphi_{00}}-1,$$

$$\tilde{\Theta}\equiv\frac{\Theta(\upsilon)}{\Theta_0}=1+\Omega\upsilon,\ \text{where}\ \Omega=\frac{\Theta_\omega}{\Theta_0}-1,$$

is applicable, in accordance with (1) we have

$$\tilde{\varphi}(\upsilon,\tilde{\tau})\equiv\frac{\varphi(\upsilon,\tilde{\tau})}{\varphi_{00}}=1+\psi\upsilon+i_0(1+\Omega\upsilon)|\tilde{\tau}|,\ \left(i_0=\frac{\Theta_0}{\varphi_{00}}\tau_\omega\right). \tag{8}$$

The case of $(\varphi_{0\omega}/\varphi_{00})>1$ and $(\Theta_\omega/\Theta_0)>1$ corresponds to heating of the liquid, while the case of $\varphi_{0\omega}/\varphi_{00})<1$ and $\Theta_\omega/\Theta_0<1$ corresponds to cooling of the liquid.

With the usual boundary conditions the equation for the dimensionless velocity over the cross section of the channel with allowance for (3) and (5) has the form

$$W_X(X,Y)=0.5\int_0^Y \tilde{\varphi}(\upsilon,\tilde{\tau})(1-Y)\,dY\left\{\int_0^1 (1-Y)\left[\int_0^Y \tilde{\varphi}(\upsilon,\tilde{\tau})(1-Y)\,dY\right]dY\right\}^{-1}. \quad (9)$$

Non-Newtonian liquids are characterized by considerable Prandtl numbers (σ) and Schmidt numbers (Sc), i.e., dynamic disturbances propagate more intensely in them than thermal (diffusional) disturbances. Therefore, under the conditions of the problem only those values of Y which are much smaller than the height of the channel are important for short channels.

Substituting (8) into (9) with allowance for (7) and neglecting (because of the smallness) the terms containing Y to more than the first power, we obtain an expression for the dimensionless velocity which is valid for the boundary region:

$$W_\omega(Y)=PY,$$
$$P=4[1+\psi+i_0(1+\Omega)]\,[1+m\psi+i_0(0.8+\Omega n)]^{-1}, \quad (10)$$
$$m=\frac{\frac{1}{6}-\frac{2}{21}K}{\frac{1}{4}-\frac{1}{6}K},\quad n=\frac{\frac{1}{7}-\frac{1}{12}K}{\frac{1}{4}-\frac{1}{6}K}.$$

In connection with the small thickness of the thermal boundary layer ($\Delta \ll 1$), when the value of Y is negligibly small compared with unity within its limits one can write, in accordance with (6), the equation for the temperature distribution near the wall:

$$0.5\,\mathrm{Pe}\,(1-K)\,Y\frac{\partial\upsilon}{\partial X}=\frac{\partial^2\upsilon}{\partial Y^2}.$$

The value of K is determined through $\tau_\omega=(1/\varphi_\omega)(\partial\omega_X/\partial y)_{y=0}$ in accordance with (10).

Thus, the cylindrical thermal boundary layer is replaced here by a flat boundary layer, which is fully admissible because of its small thickness.

The self-similar variable

$$\eta=Y\left(\frac{9X}{L}\right)^{-1/3},\quad L=0.5\,\mathrm{Pe}_0\,P\,(1-K)$$

reduces it to the ordinary equation (the primes denote derivatives with respect to η)

$$\upsilon''+3\eta^2\upsilon'=0,$$

the solution of which for the boundary conditions

$$\upsilon=1 \text{ for } Y=0,\quad \upsilon=0 \text{ for } X=0$$

has the form

$$\upsilon=1-1.12\int_0^\eta \exp(-\eta^3)\,d\eta\approx 1-G(X)\,Y,$$
$$G(X)=1.12\left(\frac{9X}{L}\right)^{-1/3}.$$

From this one can obtain an expression for the thickness of the thermal boundary layer,

$$\Delta\approx-\left(\left.\frac{\partial\upsilon}{\partial Y}\right|_{Y=0}\right)^{-1}=0.893\left(\frac{9X}{L}\right)^{1/3}. \quad (11)$$

Let us use Δ in Eq. (9),

$$W_X(X,Y)=0.5\int_0^Y \tilde{\varphi}(\upsilon,\tilde{\tau})(1-Y)\,dY\left\{\int_0^\Delta (1-Y)\left[\int_0^Y \tilde{\varphi}(\upsilon,\tilde{\tau})(1-Y)\,dY\right]\times\right.$$
$$\left.\times\, dY+\int_\Delta^1 (1-Y)\left[\int_0^\Delta \tilde{\varphi}(\upsilon,\tilde{\tau})(1-Y)\,dY+\int_\Delta^Y \tilde{\varphi}(\tilde{\tau})(1-Y)\,dY\right]dY\right\}^{-1}. \quad (12)$$

Here $\tilde{\varphi}(\tilde{\tau},\upsilon)$ is introduced with allowance for the second approximation for the temperature profile,

$$\tilde{\varphi}(\tilde{\upsilon},\tau)=1+\psi(1-G(x)Y)+i_0[1+\Omega(1-G(X)Y)](1-Y).$$

Integrating (12) and neglecting (because of smallness) the terms in the final expression for $W_X(X, Y)$ which contain Y to more than the first power, we obtain

$$W_x(X, Y)=B_1(X)Y,$$
$$B_1(X)=[1+\psi+i_0(1+\Omega)][\Delta(\psi+\Omega i_0)+0{,}25 i_0]^{-1}. \quad (13)$$

Since the thermal boundary layer is flat, from the condition of continuity we find

$$W_Y=-\int_0^Y \frac{\partial W_X}{\partial X}dY=-\frac{1}{2}\frac{\partial B_1(X)}{\partial X}Y^2.$$

Thus, Eq. (6) is reduced to the form

$$0.5\,\mathrm{Pe}_0(1-K)\left[B_1(X)Y\frac{\partial \upsilon}{\partial X}-0{,}5\frac{\partial B_1(X)}{\partial X}Y^2\frac{\partial \upsilon}{\partial Y}\right]=\frac{\partial^2\upsilon}{\partial Y^2}. \quad (14)$$

By the introduction of the variable [7]

$$\eta_1=Y\left[\exp(-S(X))\int_0^X \exp(S(X))B(X)\,dX\right]^{-1/3},$$
$$S(X)=1.5\int_0^X (B_1(X))^{-1}\frac{\partial B_1(X)}{\partial X}dX,$$
$$B(X)=0.5\,\mathrm{Pe}_0(1-K)B_1(X)$$

Eq. (14) is transformed to the ordinary equation (the primes denote derivatives with respect to η_1)

$$\upsilon''+\frac{1}{3}\eta_1^2\upsilon'=0,$$

the solution of which with the boundary conditions

$$\upsilon=1 \text{ where } Y=0,\ \upsilon=0 \text{ where } X=0$$

has the form

$$\upsilon=\int_{\eta_1}^{\infty}\exp\left(-\frac{1}{9}\eta_1^3\right)d\eta_1\left(\int_0^{\infty}\exp\left(-\frac{1}{9}\eta_1^3\right)d\eta_1\right)^{-1}. \quad (15)$$

Consequently,

$$\mathrm{Nu}_X=-2\left(\frac{\partial\upsilon}{\partial Y}\right)_{Y=0}=-2\left[\exp(-S(X))\int_0^X\exp(S(X))\,B(X)\,dX\right]^{-1/3}\frac{d\upsilon}{d\eta_1}\Big|_{\eta_1=0}. \quad (16)$$

By differentiating (15) and computing the corresponding integrals in (16), we find an expression for the local Nusselt number,

$$\mathrm{Nu}_X=1.077\left(\chi\,\mathrm{Pe}_0\frac{d}{x}\right)^{1/3}f(z), \quad (17)$$
$$f(z)=\left\{\frac{3z^{1.5}}{(z-1)^3}[0.4(z^{2.5}-1)-\frac{4}{3}(z^{1.5}-1)+2(z^{0.5}-1)]\right\}^{-1/3},$$
$$\chi=[1+\psi+i_0(1+\Omega)](1-K)[1+0.8i_0]^{-1},$$
$$Z=1+\frac{7.42(\psi+\Omega i_0)}{1+0.8i_0}\left(\frac{1+m\psi+i_0(0.8+\Omega n)}{(1-K)[1+\psi+i_0(1+\Omega)]}\frac{1}{\mathrm{Pe}_0}\frac{x}{d}\right)^{1/3}.$$

In the case when $\psi=\Omega=K=0$

$$\lim_{Z\to1}f'''(Z)\to1,$$

we have the well-known expression for the isothermal flow of structurally viscous liquids [8], and when, in addition, $i_0=0$ we have the expression for ordinary Newtonian liquids [1]. In Fig. 1 the results of an analytical calculation for ordinary liquids ($i_0=K=0$) according to Eq. (17) (curve 2: $\varphi_\omega/\varphi_0=2.5$; curve 5: $\varphi_\omega/\varphi_0=0.25$; $W_Y\neq0$) are compared for the same conditions with the tabular data of a numerical calculation by Yang Wang-tsu [9] (curve 7: $\varphi_\omega/\varphi_0=2.5$; curve 8: $\varphi_\omega/\varphi_0=0.25$; $W_Y=0$) which were obtained by an improved integral method. The slopes of curves 2 and 5 differ somewhat from the slopes of the curves in [9], which is evidently connected with the contribution of the transverse component of the velocity to the heat transfer in proportion to the increase in the thermal boundary layer. Curves 1, 3, 4, and 6 of Fig. 1 correspond to the values $\varphi_\omega/\varphi_0=12.8$, 1, 0.4, and 0.1.

The Nusselt number determined by the present method without allowance for W_Y has the form

$$\mathrm{Nu}_X=1.077\left[\frac{1+\psi}{1+5.565\psi\left(\frac{1+0.666\psi}{1+\psi}\right)^{1/3}\left(\frac{1}{\mathrm{Pe}}\frac{x}{d}\right)^{1/3}}\right]^{1/3}\cdot\left(\mathrm{Pe}\frac{d}{x}\right)^{1/3}.$$

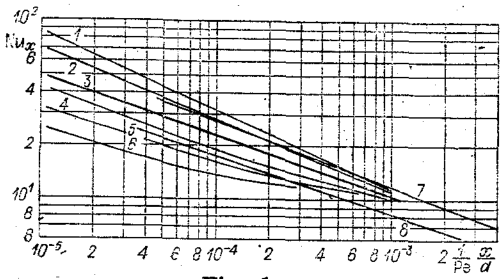

Fig. 1

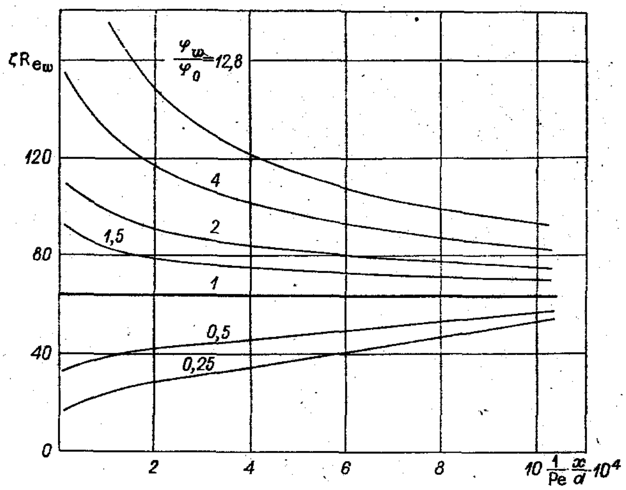

Fig. 2

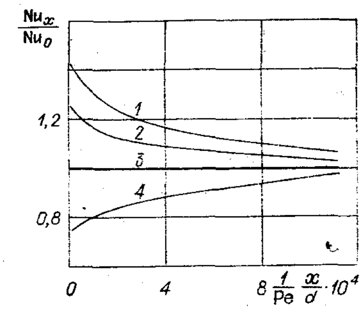

Fig. 3

The dependence $Nu_X = f[(1/Pe)(x/d)]$ plotted from this equation ($\varphi_\omega/\varphi_0 = 2.5$) coincides with that of [9].

The agreement of the results obtained by our method with the results of the numerical calculation of [9] in which nonlinear terms were taken into account in the approximation of the velocity profile indicates their negligible effect on the heat exchange in the thermal section of channels when the stream is hydrodynamically stabilized.

A marked deviation is observed at $\psi = 9$, and in the range of $(1/Pe)(x/d) \approx 10^{-3}$–$10^{-5}$ the curve is located down an average of 3.5%. In accordance with (13) the expression for the coefficient of friction for a liquid moving in a tube takes the form

$$\zeta(X) = \frac{8}{\rho\langle\omega\rangle^2}\frac{1}{\varphi_\omega}\left.\frac{\partial\omega_X}{\partial y}\right|_{y=0} = \frac{4(\Delta\psi+0.25)}{\beta_0(\Delta\Omega+0,2)}\left[\left(1+\frac{8\beta_0(\Delta\Omega+0,2)}{\mathrm{Re}_{00}(\Delta\psi+0,25)^2}\right)^{0,5}-1\right],$$

where Δ is determined by Eq. (11).

For ordinary liquids ($\Theta = 0$) when $K = 0$

$$\lim_{\beta\to 0}\zeta'(X) = \frac{16}{\mathrm{Re}_0(\Delta\psi+0,25)} = \frac{64(1+\psi)}{\mathrm{Re}_\omega}\left[1+7.42\psi\left(\frac{1+\frac{2}{3}\psi}{1+\psi}\right)^{1/3}\left(\frac{1}{\mathrm{Pe}}\frac{x}{d}\right)^{1/3}\right]^{-1}. \tag{18}$$

The results of calculation by Eq. (18) for the cases of heating ($\psi > 0$) and cooling ($\psi < 0$) of the liquid are shown in Fig. 2. The discontinuity in the solution as $(1/Pe)(x/d) \to 0$ is explained by the abrupt change in viscosity at the leading point of the start of heating.

Thus, equations are obtained from which the complexes determining the process are seen directly. The commonly used estimate of the effect of the nonisothermicity on the heat transfer and the coefficient of hydraulic resistance based on only one parameter $(\mu_0/\mu_\omega)^n$ is incomplete, and it is necessary to allow for the effect of the factor $[(1/Pe)(x/d)]^m$ of the process. The value $[(1/Pe)(x/d)]$ can make an important contribution to the isothermicity function $f(Z) = Nu_X/Nu_0$, where $Nu_0 = 1.077\ [Pe(d/x)]^{1/3}$. The results of calculations by Eq. (17) for the case of $\Theta = K = 0$ are presented in Fig. 3 (curves 1-4 correspond to the values $\varphi_\omega/\varphi_0 = 9$, 2.5, 1, and 0.4).

Equations of analogous structure are obtained for the flow of a liquid in a flat slot.

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